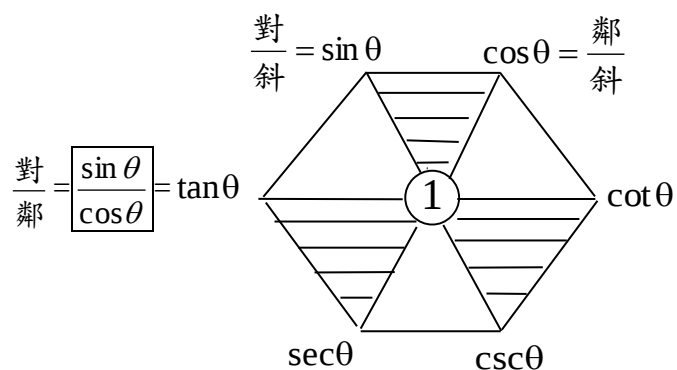
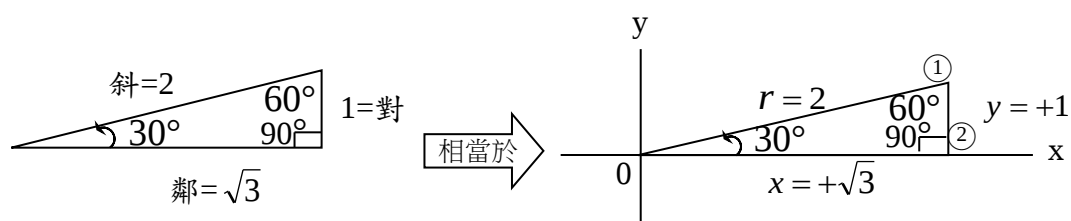


有向角的三角函數定義

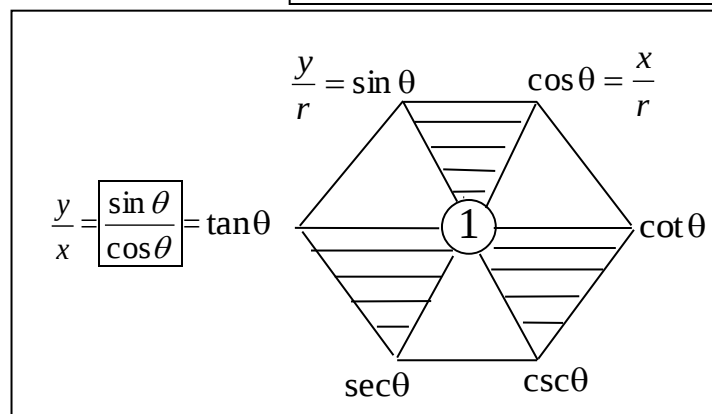
任意有向角的三角函數值應如何定義？由銳角三角函數的概念可擴充為一般有向角的情形！

任意角 θ 的三角值之作圖步驟：

- ①依角 θ 的正/負畫定終邊，
- ②在終邊上取一點，並畫一垂直於X軸的直線，
- ③標上X和Y坐標，
- ④利用坐標，求算三角值。

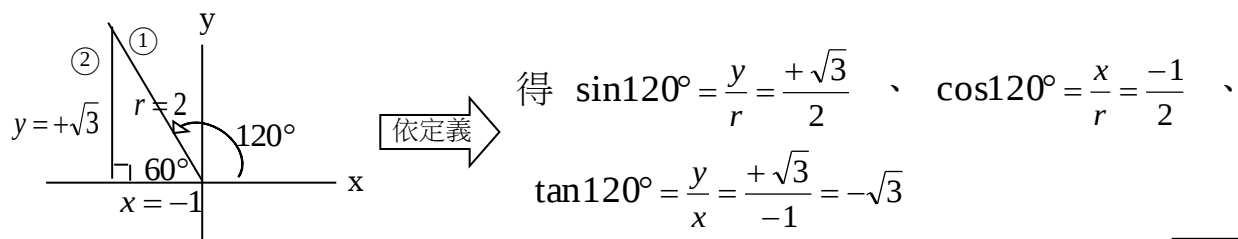


擴充定義為



例：試求 $\sin 120^\circ = ?$

解：

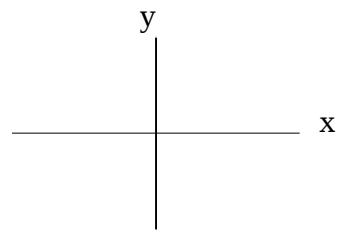
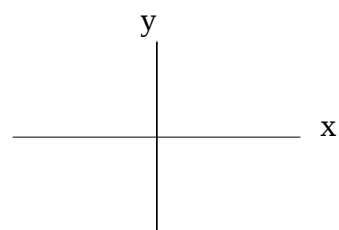
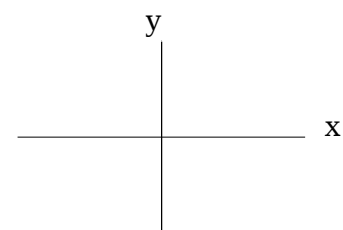


一、) 依任意角的三角函數之定義，求出指定的三角函數值：(特別角!)

① $\sin 30^\circ =$ _____

② $\cos 45^\circ =$ _____

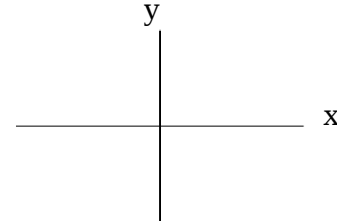
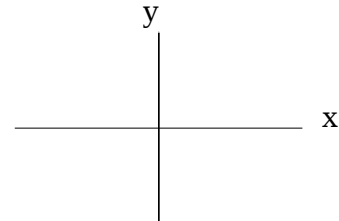
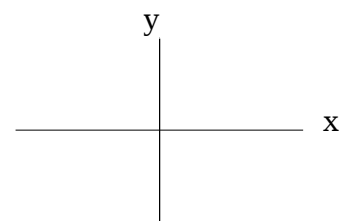
③ $\tan 60^\circ =$ _____



④ $\cot 120^\circ =$ _____

⑤ $\sec 135^\circ =$ _____

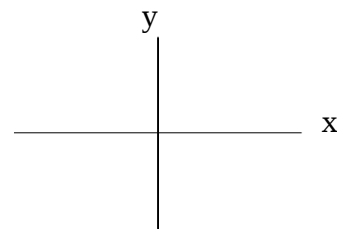
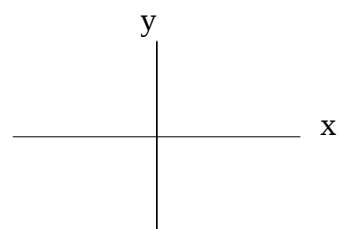
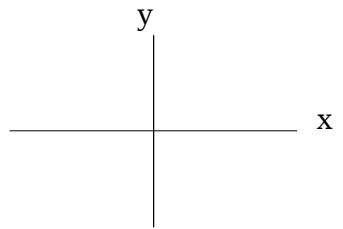
⑥ $\csc 150^\circ =$ _____



⑦ $\sin 210^\circ =$ _____

⑧ $\cos 225^\circ =$ _____

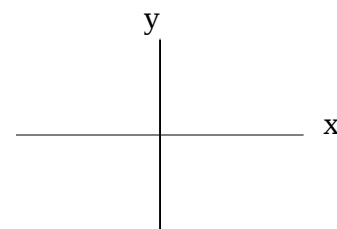
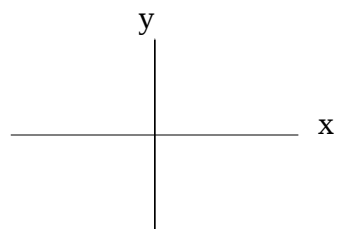
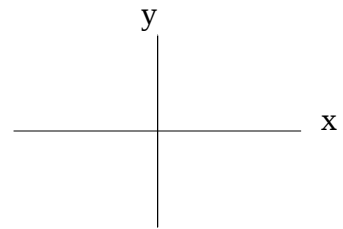
⑨ $\tan 240^\circ =$ _____



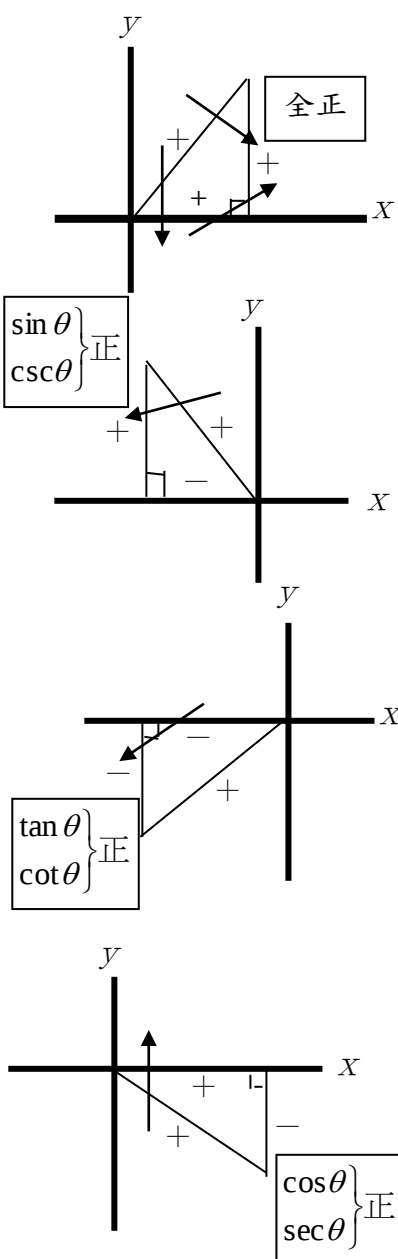
⑩ $\cot 300^\circ =$ _____

⑪ $\sec 315^\circ =$ _____

⑫ $\csc 330^\circ =$ _____

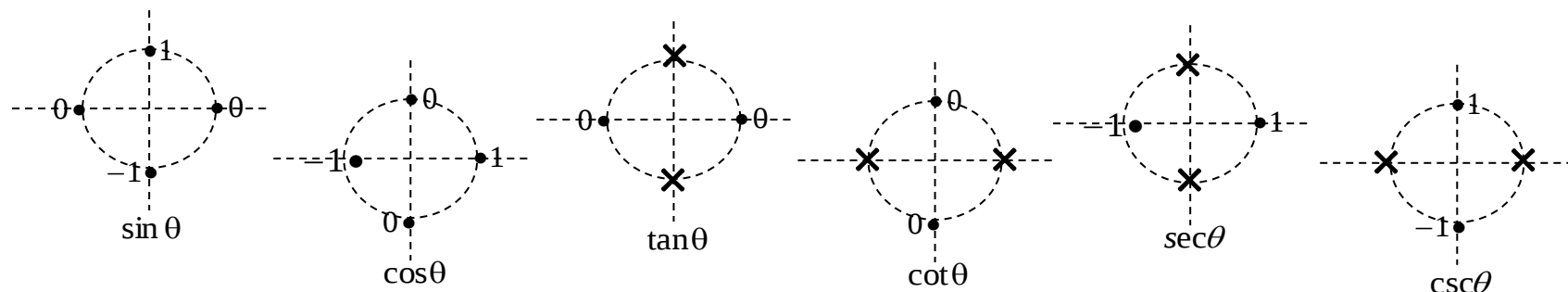


三角函數的正/負性質：

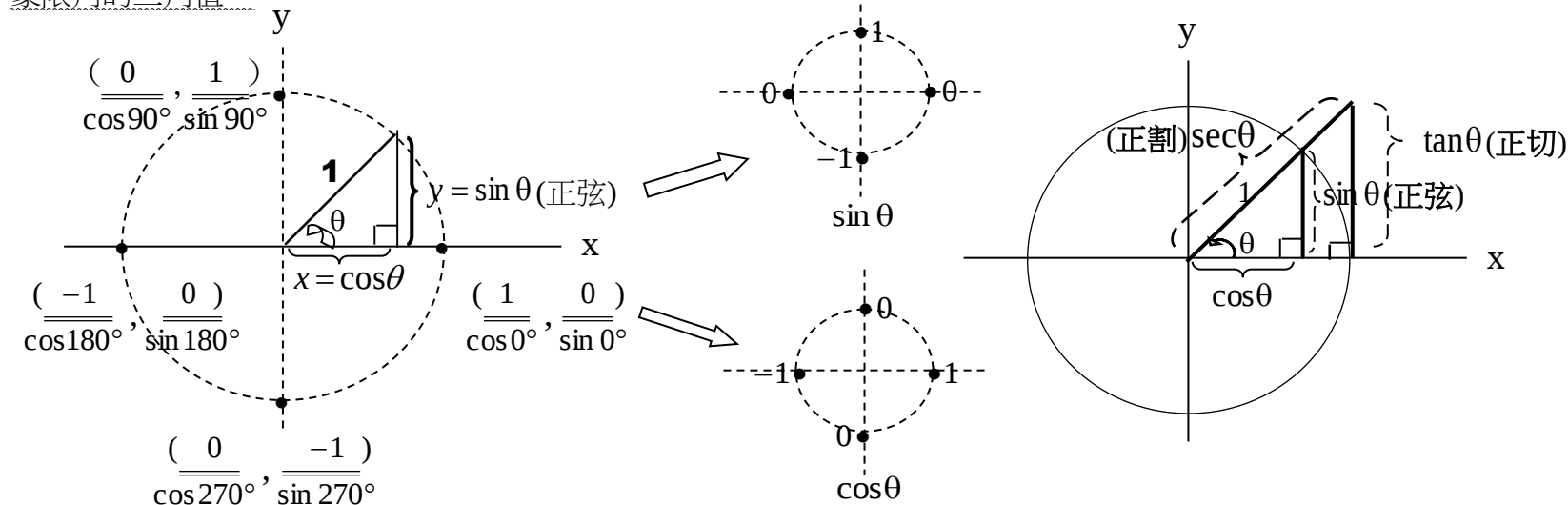


二、) 象限角的三角值為何？(特別角!)

- ① $\sin 0^\circ =$ _____ ⑤ $\cos 0^\circ =$ _____ ⑨ $\tan 0^\circ =$ _____ ⑬ $\cot 0^\circ =$ _____ ⑰ $\sec 0^\circ =$ _____ ⑳ $\csc 0^\circ =$ _____
- ② $\sin 90^\circ =$ _____ ⑥ $\cos 90^\circ =$ _____ ⑩ $\tan 90^\circ =$ _____ ⑭ $\cot 90^\circ =$ _____ ⑱ $\sec 90^\circ =$ _____ ㉒ $\csc 90^\circ =$ _____
- ③ $\sin 180^\circ =$ _____ ⑦ $\cos 180^\circ =$ _____ ⑪ $\tan 180^\circ =$ _____ ⑮ $\cot 180^\circ =$ _____ ⑲ $\sec 180^\circ =$ _____ ㉓ $\csc 180^\circ =$ _____
- ④ $\sin 270^\circ =$ _____ ⑧ $\cos 270^\circ =$ _____ ⑫ $\tan 270^\circ =$ _____ ⑯ $\cot 270^\circ =$ _____ ㉔ $\sec 270^\circ =$ _____ ㉕ $\csc 270^\circ =$ _____



象限角的三角值：



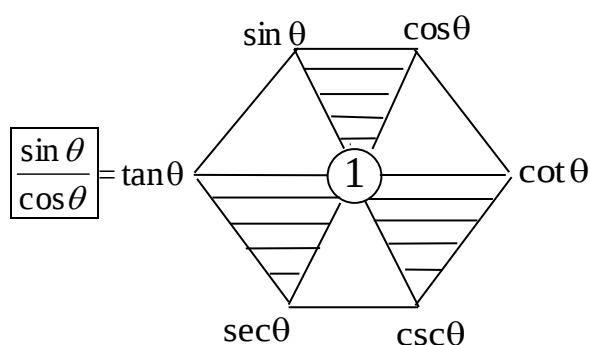
四、) 三角函數的性質：

(1) 倒數公式：

$$① \csc \theta = \frac{1}{\sin \theta}$$

$$② \sec \theta = \frac{1}{\cos \theta}$$

$$③ \cot \theta = \frac{1}{\tan \theta}$$



(2) 平方公式：

$$① \sin^2 \theta + \cos^2 \theta = 1 \quad (\text{即 } \sin^2 \theta = 1 - \cos^2 \theta; \cos^2 \theta = 1 - \sin^2 \theta)$$

$$② 1 + \tan^2 \theta = \sec^2 \theta \quad (\text{即 } \tan^2 \theta = \sec^2 \theta - 1; \sec^2 \theta - \tan^2 \theta = 1)$$

$$③ 1 + \cot^2 \theta = \csc^2 \theta \quad (\text{即 } \cot^2 \theta = \csc^2 \theta - 1; \csc^2 \theta - \cot^2 \theta = 1)$$

(3) 常用公式：

$$① (\sin \theta + \cos \theta)^2 = 1 + 2 \sin \theta \cdot \cos \theta; (\sin \theta - \cos \theta)^2 = 1 - 2 \sin \theta \cdot \cos \theta$$

$$② (\sin \theta \pm \cos \theta)^3 = (\sin \theta \pm \cos \theta)^3 \pm 3 \cdot \sin \theta \cdot \cos \theta \cdot (\sin \theta \pm \cos \theta)$$

$$\text{註：} (\alpha \pm \beta)^3 = (\alpha \pm \beta)^3 \pm 3 \cdot \alpha \cdot \beta \cdot (\alpha \pm \beta)$$

$$③ \tan \theta + \cot \theta = \frac{1}{\sin \theta \cdot \cos \theta} \quad ④ \sec \theta \pm \csc \theta = \frac{\sin \theta \pm \cos \theta}{\sin \theta \cdot \cos \theta}$$